COSMOLOGY OF RANDALL-SUNDRUM MODELS*

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There are many interesting issues in the brane world with a large/warped extra dimension. We focus on the cosmological aspects. We review the cosmological solutions of the brane world and how the conventional four-dimensional cosmology is recovered by including the effect of stabilization. Its implications on the mass hierarchy and the cosmological constant are discussed.

1 Introduction

For past two years, many particle physicists and cosmologists were excited by the development of two ideas, the brane world and the warped extra dimensions, both of which are based on the existence of extra dimensions. The basic idea of the brane world is that standard model particles are localized on a (3+1)-dimensional brane (or a set of branes) embedded in a higher dimensional spacetime, while gravity propagates in the whole bulk [1,2]. The warped extra dimension assumes that the background metric is curved along the extra dimensions, mainly due to the negative bulk cosmological constant [3,4]. Why are these two ideas so exciting? They have brought us fresh views and perspectives in gravity, cosmology, particle physics and string theory. We have seen many interesting issues discussed so far, such as the localization of gravity, the gauge hierarchy problem, the cosmological constant problem and self-tuning models [5], the construction of supersymmetric RS models and the role of supersymmetry [6], the connection to string theory or Horava-Witten model and warped compactification, the interpretation in light of AdS/CFT holographic duality [7], the collider signatures of the KK modes and the radion, etc [8].

In this talk, we focus on the cosmological aspect of the two ideas, mostly in five dimensional models with one extra dimension. There has been much interest in this because the five dimensional nature of gravity and the brane setup might lead to the non-conventional cosmology even at low temperature as well as at high temperature above TeV scale. It was found that the Friedmann equation of the brane shows a $H \propto \rho$ behavior and has an additional dark radiation term [9]. There was also a difficulty concerning the negative

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tension brane, and it was not very clear what happens at temperatures above TeV, which can alter the early universe cosmology including inflation.

However, in the early models, a few important ingredients are not addressed, such as the mechanism for the localization of fields on the brane, the stabilization of the brane systems [10] and the way to achieve necessary fine tunings of parameters. They inevitably involve bulk matter and dynamics, and can change the whole picture, for example, changing the exponential warp factor to power law. For the cosmological consequences of brane world models, taking the stabilization into account is found to be crucial [11,12,13]. The inclusion of the effect of stabilization recovers the conventional FRW cosmology at temperatures below TeV.

The talk is organized as follows. First, we derive the effective four-dimensional brane equations to see the generalities of brane dynamics, though its usefulness is limited by the lack of the knowledge of bulk effects. Then, we try to solve the five dimensional equations with an appropriate ansatz. The solution can be obtained in very restricted cases, but a framework can be found where we can study brane cosmology with the effect of stabilization taken into account. This is done through the $\hat{T}^5_5$ component which is adjusted to stabilize the extra dimension in the presence of brane matter. Based on this, we analyze the background spacetime where we also discuss the mass scales and the hierarchy problem, and one and two brane models in turn.

2 Effective four-dimensional equations on the brane

2.1 Framework

We consider the five-dimensional spacetime with coordinates (τ, x^i, y), and 3-branes embedded in it. The action describing our framework is

$$\begin{aligned} S = &\int_M d^5x\sqrt{-\hat{g}}\left[\frac{M^3}{2}\hat{R} - \Lambda_b + \mathcal{L}_{bM}\right] \\ &+ \sum_i \int d^4x\sqrt{-g^{(i)}}\left[-\Lambda_i + \mathcal{L}_{iM}\right] \end{aligned} \tag{1}$$

where M is the fundamental gravitational scale of the model, Λ_b and Λ_i represent the bulk cosmological constant and brane tensions, respectively. $\mathcal{L}_{bM}$ and $\mathcal{L}_{iM}$ are Lagrangians for the bulk fields and for the fields localized in the branes.

To investigate the role of bulk and bulk fields in brane dynamics [14] we introduce a bulk scalar $\hat{\Phi}$, with $\mathcal{L}_{bM} = \frac{1}{2}(\hat{\partial}\hat{\Phi})^2$. We also allow that Λ_b, Λ_i and $\mathcal{L}_i$ can be functions of $\hat{\Phi}$. The bulk Einstein equations obtained from the

action (1) are

$$\hat{G}_{MN} = \hat{T}_{MN} = \hat{\partial}_M \hat{\Phi} \hat{\partial}_N \hat{\Phi} - \hat{g}_{MN} \left[\frac{1}{2} (\hat{\partial} \hat{\Phi})^2 + \Lambda_b(\hat{\Phi}) \right], \tag{2}$$

and the bulk scalar equation is

$$\hat{\Box} \hat{\Phi} - \frac{d\Lambda_b(\hat{\Phi})}{d\hat{\Phi}} = 0. \tag{3}$$

The existence of branes imposes the junction conditions on the above bulk equation, giving discontinuities in the first derivatives across the branes of metric and scalar field. First let us proceed the general setup. A brane can be described by the normal vector n_M. Then the induced metric on the brane is given by $g_{MN} = \hat{g}_{MN} - n_M n_N$, while the extrinsic curvature by $K_{MN} = \partial_M n_N$. The junction conditions are

$$[K_{\mu\nu}] = -\frac{1}{2M^3} \left(T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T \right), \tag{4}$$

$$[n_M \partial^M \hat{\Phi}] = \frac{1}{2M^3} \left(\frac{d\Lambda_i}{d\hat{\Phi}} + \frac{\delta \mathcal{L}_{iM}}{\delta \hat{\Phi}} \right), \tag{5}$$

where the bracket in the left hand side means the difference across the brane and $T_{\mu\nu}$ is the energy momentum tensor of brane matter, *i.e.*, Λ_i and $\mathcal{L}_{iM}$.

Suppose that we are localized on a certain brane and want to study the brane dynamics as observed by us. We may take two different approaches. The first approach is to derive the effective 3-brane equations localized on our brane. The second is to directly solve the whole bulk equations. In this section, we follow the first approach. The second will be dealt with in the next section.

2.2 The effective 3-brane Einstein equation

To derive the effective 3-brane equations, we need to know the extrinsic curvature and the intrinsic curvature of our brane in terms of the bulk metric $\hat{g}_{MN}$ and the normal vector n_M, which are provided by the Codacci equation

$$\partial_M K^M_\mu - \partial_\mu K = g^M_\mu G_{MN} n^N, \tag{6}$$

and the Gauss equation

$$G_{\mu\nu} = \frac{2}{3} \left[G_{MN} g^M_\mu g^N_\nu + \left(G_{MN} n^M n^N - \frac{1}{4} G \right) g_{\mu\nu} \right] + K K_{\mu\nu} - K^M_\mu K_{M\nu} - \frac{1}{2} \left(K^2 - K_{MN} K^{MN} \right) g_{\mu\nu} - E_{\mu\nu}, \tag{7}$$

where $E_{\mu\nu} = C_{MNOP} n^M n^O g_\mu^N g_\nu^P$ and C_{MNOP} is the bulk Weyl tensor.

Now we take y as the Gaussian normal coordinates and impose Z_2 symmetry, $y \sim -y$. We expand $\hat{\Phi}$ around the brane,

$$\hat{\Phi}(x, y) = \phi(x) + \Phi_1(x)|y| + \frac{1}{2}\Phi_2(x) y^2 + \mathcal{O}(y^3). \tag{8}$$

Then from the bulk equations (2) and (3), and the junction conditions (4) and (5), we obtain the equation for ϕ

$$\Box\phi - \frac{d\Lambda_b}{d\phi} = -\frac{T}{12M^3}\left(\frac{d\Lambda}{d\phi}\right) - \Phi_2, \tag{9}$$

and the four-dimensional effective Einstein equation for the brane [14]

$$G_{\mu\nu} = \Lambda_{\rm eff}(\phi) g_{\mu\nu} + \frac{\Lambda(\phi)}{6M^3} T_{\mu\nu} + \frac{1}{M^6}\Pi_{\mu\nu} - E_{\mu\nu} + \frac{2}{3M^3}\hat{T}_{\mu\nu}(\phi), \tag{10}$$

where

$$\Lambda_{\rm eff}(\phi) = \frac{1}{2M^3}\left[\Lambda_b + \frac{\Lambda^2}{6M^3} - \frac{1}{8}\left(\frac{d\Lambda}{d\phi}\right)^2\right], \tag{11}$$

$$\Pi_{\mu\nu} = -\frac{1}{4} T_{\mu\lambda} T_\nu^\lambda + \frac{1}{12} T T_{\mu\nu} + \frac{1}{8} g_{\mu\nu} T_{\lambda\rho} T^{\lambda\rho} - \frac{1}{24} g_{\mu\nu} T^2, \tag{12}$$

$$\hat{T}_{\mu\nu}(\phi) = \partial_\mu\phi\partial_\nu\phi - \frac{5}{8} g_{\mu\nu}(\partial\phi)^2. \tag{13}$$

The first three terms in the right hand side of (10) deal with the sources on the brane. The first two terms are same as four-dimensional Einstein gravity, if we identify the four-dimensional Planck mass

$$M_P^{-2} = \frac{\Lambda(\phi)}{6M^6}. \tag{14}$$

The third term gives a correction quadratic in energy-momentum tensor. The last two terms in Eq. (10) are bulk effect terms, which reflect the existence of bulk in brane dynamics. They are inputs from bulk dynamics and not determined in the brane equations. The brane equations (9) and (10) are not closed equations. In this sense, they would not be very useful without the knowledge of the bulk effect terms. However, they reveal a most general structure of the brane equations. In this regard, we note that the Planck mass (14) seems at first to be determined solely by the brane tension. This seems strange because the graviton (and its zero mode) comes from the bulk fields. Therefore there must be some correlation between the brane tension and bulk dynamics, which are incorporated in the brane equations through these bulk effect terms. We will see this in the following sections where the bulk solutions are treated.

3 Five-dimensional cosmological solutions

3.1 Framework

In this section, we investigate the cosmological bulk solutions of five-dimensional models. For the two brane models, the fifth dimension y is assumed to be an orbifold S^1/Z_2 with $y \sim y+1$ and $y \sim -y$ identified. Two 3-branes reside at two fixed points (boundaries) $y = 0, \frac{1}{2}$.

Since we are interested in the cosmological solution, we consider the metric where the 3-dimensional spatial section is homogeneous and isotropic.

$$ds^2 = -n^2(\tau,y)d\tau^2 + a^2(\tau,y)\gamma_{ij}dx^i dx^j + b^2(\tau,y)dy^2, \tag{15}$$

where γ_{ij} is the 3-dimensional homogeneous and isotropic metric, and we will use $K = -1, 0, +1$ to represent its spatial curvature. Einstein equations are given by $\hat{G}_{MN} = (1/M^3)\hat{T}_{MN}$ where

$$\hat{G}_{00} = 3\left\{\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right) - \frac{n^2}{b^2}\left[\frac{a''}{a}+\frac{a'}{a}\left(\frac{a'}{a}-\frac{b'}{b}\right)\right] + K\frac{n^2}{a^2}\right\} \tag{16}$$

$$\begin{aligned}\hat{G}_{ij} = &-\frac{a^2}{n^2}\gamma_{ij}\left\{2\frac{\ddot{a}}{a}+\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}-2\frac{\dot{n}}{n}\right)+\frac{\ddot{b}}{b}+\frac{\dot{b}}{b}\left(2\frac{\dot{a}}{a}-\frac{\dot{n}}{n}\right)\right\} - K\gamma_{ij}\\ &+\frac{a^2}{b^2}\gamma_{ij}\left\{2\frac{a''}{a}+\frac{n''}{n}+\frac{a'}{a}\left(\frac{a'}{a}+2\frac{n'}{n}\right)-\frac{b'}{b}\left(2\frac{a'}{a}+\frac{n'}{n}\right)\right\}\end{aligned} \tag{17}$$

$$\hat{G}_{55} = 3\left\{-\frac{b^2}{n^2}\left[\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}-\frac{\dot{n}}{n}\right)+\frac{\ddot{a}}{a}\right]+\frac{a'}{a}\left(\frac{a'}{a}+\frac{n'}{n}\right)\right\} \tag{18}$$

$$\hat{G}_{05} = 3\left(\frac{n'}{n}\frac{\dot{a}}{a}+\frac{a'}{a}\frac{\dot{b}}{b}-\frac{\dot{a}'}{a}\right) \tag{19}$$

$$\hat{T}^M_N = \mathrm{diag}[-\hat{\rho},\hat{p},\hat{p},\hat{p},\hat{p}_5] + \sum_{i=0,\frac{1}{2}}\frac{\delta(y_i)}{b}\mathrm{diag}[-\rho_i,p_i,p_i,p_i,0] \tag{20}$$

In addition to Einstein equations, we use the energy-momentum conservation equation, $\partial_M \hat{T}^M_N = 0$

$$\frac{d\hat{\rho}}{d\tau} + 3(\hat{\rho}+\hat{p})\frac{\dot{a}}{a} + (\hat{\rho}+\hat{p}_5)\frac{\dot{b}}{b} = 0, \tag{21}$$

$$\hat{p}_5' + \hat{p}_5\left(\frac{n'}{n}+3\frac{a'}{a}\right) + \frac{n'}{n}\hat{\rho} - 3\frac{a'}{a}\hat{\rho} = 0. \tag{22}$$

Brane sources in Eq. (20) can be converted to boundary conditions (Junction conditions): n, a and b must be continuous and n', a' are discontinuous

due to boundary sources by

$$\frac{1}{b}\frac{n'}{n}\bigg|_{y_i-}^{y_i+} = \frac{2\rho_i + 3p_i}{3M^3}, \qquad \frac{1}{b}\frac{a'}{a}\bigg|_{y_i-}^{y_i+} = -\frac{\rho_i}{3M^3}. \tag{23}$$

3.2 Five-dimensional spacetime with the bulk cosmological constant and brane tensions

First, we consider the five-dimensional spacetime supported by the negative bulk cosmological constant $\hat{\rho} = -\hat{p} = \Lambda_b < 0$ and brane tensions $\rho_i = -p_i = \Lambda_i$. We define the parameters

$$k = \left(\frac{-\Lambda_b}{6M^3}\right)^{1/2}, \quad k_i = \frac{\Lambda_i}{6M^3} \tag{24}$$

In general cases with $k_0, -k_{1/2} \geq k$, the metric is given by [15]

$$ds^2 = \frac{-d\tau^2 + \delta_{ij}dx^i dx^j + (k\tau b_0)^2 dy^2}{\left[k\tau \sinh(kb_0|y| + c_0) + g_0\right]^2} \tag{25}$$

where b_0 and c_0 are determined by

$$k\cosh(c_0) = k_0, \qquad k\cosh(\frac{1}{2}kb_0 + c_0) = -k_{\frac{1}{2}}. \tag{26}$$

The metric (25) describes a slice of AdS$_5$ space inflating both in extra dimension and in spatial dimensions. We note two special cases. If we have a fine tuning $(-k_2 - k)/(k_1 - k) = e^{kb_o}(-k_2 + k)/(k_1 + k)$, we obtain a inflationary solution with static extra dimension [16]

$$ds^2 = \frac{-d\tau^2 + e^{2k\tau}\delta_{ij}dx^i dx^j + b_0^2 dy^2}{\sinh^2(kb_0|y| + c_0)} \tag{27}$$

With two fine tunings $k = k_0 = -k_{\frac{1}{2}}$, we can get a static solution, the Randall-Sundrum model [4]

$$ds^2 = e^{-2kb_0|y|}\eta_{\mu\nu}dx^\mu dx^\nu + b_0^2 dy^2 \tag{28}$$

This model attracted much attention because it convert the gauge hierarchy problem into a geometric problem. The four-dimensional Planck scale in this model is given by $M_P^2 = (M^3/k)[1 - e^{-kb_0}]$. Any mass parameter m_0 on the visible brane corresponds to a physical mass $m = m_0 e^{-\frac{1}{2}kb_0}$, which we identify as the weak scale. Hence the large hierarchy between the Planck scale and the weak scale $M_P/M_W \sim 10^{16}$ can be explained by a warp factor $e^{-\frac{1}{2}kb_0}$ with $\frac{1}{2}kb_0 \sim 37$. The property is not spoiled by R^2 corrections, if R^2 corrections are given by Gauss-Bonnet interactions [17].

Note that the space described by the metric (25) is locally AdS_5, because it shares the same bulk equation. The metric (25) can be transformed to (28) by a coordinate transformation

$$e^{-kb_{\mathrm{RS}}y_{\mathrm{RS}}} = k\tau \sinh(kb_0 y + c_0) + g_0, \quad \tau_{\mathrm{RS}} = \tau \cosh(kb_0 y + c_0). \tag{29}$$

Therefore, we can say that above metrics are the slices of AdS_5 with different boundary geometries.

3.3 Cosmological solutions with static extra dimension

Let us turn to the more interesting situation where the matter is added on the brane(s) and possibly in the bulk. But we require the extra dimension to be static, that is $\dot{b} = 0$. This requires a fine tuning between matter densities. For the bulk energy-momentum, we assume

$$\hat{\rho} = \Lambda_b, \quad \hat{p} = -\Lambda_b, \quad \hat{p}_5 = -\Lambda_b + p_5(\tau, y). \tag{30}$$

where $\Lambda_b < 0$. The form of $p_5(\tau, y)$ is constrained by Eq. (22) to be

$$p_5(\tau, y) = \frac{\tilde{p}_5(\tau)}{n(\tau, y) a^3(\tau, y)} \tag{31}$$

With the gauge fixing $n(\tau, y = 0) = 1$, we obtain the solution [18,13]

$$a^2(\tau, y) = a_0^2 \left[\left(1 + \frac{\dot{a}_0^2 + K}{2k^2 a_0^2} \right) \cosh(2kby) - \frac{\dot{a}_0^2 + K}{2k^2 a_0^2} \right.$$
$$\left. \pm \left(1 + \frac{1}{k^2} \left[\frac{\dot{a}_0^2 + K}{a_0^2} + \frac{C}{a_0^4} \right] \right)^{1/2} \sinh(2kby) \right], \tag{32}$$

and $n(\tau, y) = \dot{a}(\tau, y)/\dot{a}_0(\tau)$. Here $a_0(\tau) \equiv a(\tau, y = 0)$ and $C(\tau)$ is determined by $\tilde{p}_5(\tau)$ up to a constant through the equation

$$\dot{C} = \frac{2\dot{a}_0(\tau)}{3M^3} \, \tilde{p}_5(\tau). \tag{33}$$

$a_0(\tau)$ is fixed by the boundary condition. Suppose that a brane with the energy density ρ_0 is placed at $y = 0$, and assume Z_2 symmetry $y \sim -y$. The junction condition (23) at the brane gives the evolution equation for $a_0(\tau)$

$$\left(\frac{\dot{a}_0}{a_0} \right)^2 + \frac{K}{a_0^2} = -k^2 + \left(\frac{\rho_0}{6M^3} \right)^2 - \frac{C}{a_0^4} \tag{34}$$

If we further assume that we are living on the brane, this equation is nothing but the Friedmann equation of our universe. However, in the simple case

where $\Lambda_b = 0$ and $p_5 = 0$, this equation differs from the four-dimensional Friedmann equation in two aspects. First, the Hubble parameter $H = \dot{a}_0/a_0$ is proportional to ρ_0 instead of $\sqrt{\rho_0}$. Second, there is a C/a_0^4 term which looks like a radiation term. This means that we can have dynamic solution without any matter on the brane or in the bulk. This term seems to have an interesting interpretation in view of AdS/CFT [7]. Both of these alter the big bang nucleosynthesis result: $H \propto \rho_0$ is not compatible and C is required to be small enough.

If we consider the negative bulk cosmological constant together with the positive brane tension, $\rho_0 = \Lambda_0 + \rho_{0M}$, the above equation is written as

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{K}{a_0^2} = (k_0^2 - k^2) + \frac{\Lambda_0}{18M^6}\rho_{0M} + \frac{1}{36M^6}\rho_{0M}^2 - \frac{C}{a_0^4}. \qquad (35)$$

The four-dimensional Friedmann equation is obtained for $k_0^2 - k^2 = 0$, $C = 0$, $\rho_{0M} \ll \Lambda_0$. We can obtain a viable cosmology for the positive tension brane attached to the infinite size extra dimension. The effective cosmological constant is given by $\Lambda_{\rm eff} \propto (k_0^2 - k^2)$ At high energy/temperature, that is, in the very early universe ρ_{0M}^2 term dominates and results in very interesting consequences [19].

4 The stabilized RS models

4.1 Stabilization by balanced bulk matter

In the previous section, we saw that the brane at $y = 0$ fixes the whole bulk solution. If we have the second brane at $y = \frac{1}{2}$, the junction condition at $y = \frac{1}{2}$ gives

$$\bar{\rho}_{\frac{1}{2}} = \frac{\sinh(kb) + \frac{1}{2}(\bar{\rho}_0^2 - \bar{C} - 1)\sinh(kb) - \bar{\rho}_0\cosh(kb)}{\cosh(kb) + \frac{1}{2}(\bar{\rho}_0^2 - \bar{C} - 1)(\cosh(kb) - 1) - \bar{\rho}_0\sinh(kb)}, \qquad (36)$$

where

$$\bar{\rho}_i = \frac{\rho_i}{6M^3k}\ (i = 0, \tfrac{1}{2}), \quad \bar{C} = \frac{C}{k^2a_0^4}. \qquad (37)$$

This is a constraint between energy densities ρ_i on two branes. The reason why this constraint is necessary is obvious. We used the ansatz with the static extra dimension, which is not the general case for the two brane model. But the (almost) static extra dimension is required from the phenomenological view point. We need a stabilization mechanism to make the extra dimension static.

Here we consider a simple modeling of how the stabilization mechanism works [13,11]. It must involve some bulk dynamics. We suppose that it is through the role of C. The basic idea is that the back reaction by the stabilization mechanism to the brane energy densities which, if left alone, would destabilize the extra dimension, induces the bulk energy momentum $\hat{T}^5_5$ which forces C fitted to the constraint (36) through (33) and keep $\dot{b} = 0$. The constraint (36) can be solved to give the necessary C

$$\bar{C} = (\bar{\rho}_0^2 - 1) - 2\,\frac{\bar{\rho}_0 + \bar{\rho}_{\frac{1}{2}} - (1 + \bar{\rho}_0\bar{\rho}_{\frac{1}{2}})\tanh(kb)}{\tanh(kb)\{1 - \bar{\rho}_{\frac{1}{2}}\tanh(kb/2)\}}, \tag{38}$$

and the bulk energy-momentum component p_5 through

$$p_5(\tau, y) = \frac{6M^3}{4a^3(\tau, y)\dot{a}(\tau, y)}\,\dot{C}. \tag{39}$$

Inserting (38) into (34), we obtain the 3-brane Friedmann equation for the stabilized two brane system

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{K}{a_0^2} = 2k^2\,\frac{\bar{\rho}_0 + \bar{\rho}_{\frac{1}{2}} - (1 + \bar{\rho}_0\bar{\rho}_{\frac{1}{2}})\tanh(kb)}{\tanh(kb)\{1 - \bar{\rho}_{\frac{1}{2}}\tanh(kb/2)\}}. \tag{40}$$

We have two comments here. First, in general it is expected that the stabilization mechanism induces $\hat{T}^i_i$ as well as $\hat{T}^5_5$, and b may not be strictly static but be shifted somewhat as ρ_i changes in time. Considering $\hat{T}^5_5$ only and requiring strictly static b seems to give a limit of infinitely steep stabilization potential. It is of course unrealistic, but gives the leading behaviors of RS models with a certain, unknown stabilization mechanism. Second, we can see from (39) that the induced p_5 does not fix the additive constant of C. This cannot be controlled by $\hat{T}^5_5$, but required to vanish to satisfy the constraint. This may require another mechanism behind. Actually non-vanishing C implies the breakdown of conformal symmetry in the bulk, and there might be a connection between the stabilization mechanism which requires non-trivial C and the conformal symmetry breaking.

Now we rephrase the metric for the stabilized RS model

$$ds^2 = -n^2(\tau, y)d\tau^2 + a^2(\tau, y)\delta_{ij}dx^i dx^j + b^2(\tau, y)dy^2 \tag{41}$$

where

$$\begin{aligned} b(\tau, y) &= b = \text{constant}, \\ n(\tau, y) &= \frac{\dot{a}(\tau, y)}{\dot{a}(\tau, 0)}, \end{aligned}$$

$$a(\tau,y)=a_0(\tau)\Bigg[\cosh(2kby)-\bar{\rho}_0\sinh(2kby)$$
$$+\frac{(\bar{\rho}_0+\bar{\rho}_{\frac{1}{2}})\cosh(kb)-(1+\bar{\rho}_0\bar{\rho}_{\frac{1}{2}})}{\sinh(kb)-\bar{\rho}_{\frac{1}{2}}\{\cosh(kb)-1\}}\{\cosh(2kby)-1\}\Bigg]^{1/2} \quad (42)$$

and $a_0(\tau)$ satisfies (40).

4.2 The static five-dimensional spacetime: The mass hierarchy and the cosmological constant

With the metric in (41) and (42), we first consider the case where there is no matter and only the bulk cosmological constant and the brane tensions are involved. The metric is now given by $a(\tau,y)=a_0(\tau)n(y)$ and

$$n(y)=\Bigg[\cosh(2kby)-\bar{k}_0\sinh(2kby)$$
$$+\frac{(\bar{k}_0+\bar{k}_{\frac{1}{2}})\cosh(kb)-(1+\bar{k}_0\bar{k}_{\frac{1}{2}})}{\sinh(kb)-\bar{k}_{\frac{1}{2}}\{\cosh(kb)-1\}}\{\cosh(2kby)-1\}\Bigg]^{1/2}. \quad (43)$$

From (38) and (39), the balanced p_5 is found to be

$$p_5(y)=\frac{6M^3k^2}{n(y)^4}\left[(\bar{k}_0^2-1)-2\frac{(\bar{k}_0+\bar{k}_{\frac{1}{2}})-(1+\bar{k}_0\bar{k}_{\frac{1}{2}})\tanh(kb)}{\tanh(kb)\{1-\bar{k}_{\frac{1}{2}}\tanh(kb/2)\}}\right]. \quad (44)$$

The scale factor $a(\tau,y)$ undergoes inflation, and the Hubble parameter can be defined independently of y because $H=\dot{a}(\tau,y)/a(\tau,y)=\dot{a}_0(\tau)/a_0(\tau)$.

The static background spacetime is obtained when we make a fine tuning to satisfy the condition for the vanishing cosmological constant

$$(\bar{k}_0+\bar{k}_{\frac{1}{2}})-(1+\bar{k}_0\bar{k}_{\frac{1}{2}})\tanh(kb)=0. \quad (45)$$

With this condition, (43) and (44) are simplified to

$$n(y)=\left[\cosh(2kby)-\bar{k}_0\sinh(2kby)\right]^{1/2}, \quad (46)$$
$$p_5(y)=\frac{6M^3(k_0^2-k^2)}{n(y)^4}. \quad (47)$$

We can identify the four-dimensional Planck scale in two ways. Firstly, we can get it from the 4-dimensional effective theory which is obtained by integrating

the action over the extra dimension,

$$M_P^2 = M^3 \int_{-\frac{1}{2}}^{\frac{1}{2}} b\,dy\,n(y)^2 = \frac{M^3}{k}\left[\sinh(kb) - \bar{k}_0\{\cosh(kb) - 1\}\right]. \qquad (48)$$

Secondly, we can deduce it from the 3-brane Friedmann equation (40), which leads to

$$M_P^2 = \frac{M^3}{k}\,\frac{\tanh(kb)\{1 - \bar{k}_{\frac{1}{2}}\tanh(kb/2)\}}{1 - \bar{k}_{\frac{1}{2}}\tanh(kb)} \qquad (49)$$

The two derived Planck scales (48) and (49) coincide under the condition (45), that is, when the cosmological constant vanishes.

Let us consider the gauge hierarchy problem in this background spacetime. Physical mass scale at two branes are given by [11]

$$M_W = Mn(\tau, 0) = M, \qquad (50)$$

$$M_H = Mn(\tau, \frac{1}{2}) = M\left[\cosh(kb) - \bar{k}_0\sinh(kb)\right]^{1/2}. \qquad (51)$$

Note that we placed the visible brane at $y = 0$ and identified the physical mass scale of the visible brane as the weak scale. The physical mass scale on the hidden brane, called the hidden scale here, is in general different from the Planck scale. Now the ratio of the electroweak scale and the Planck scale is

$$\frac{M_P^2}{M_W^2} = \left(\frac{M}{k}\right)\left[\sinh(kb) - \bar{k}_0\left\{\cosh(kb) - 1\right\}\right] \approx 10^{32}. \qquad (52)$$

The condition for the vanishing cosmological constant and the solution to the gauge hierarchy problem impose two conditions among four parameters k, k_0, $k_{\frac{1}{2}}$ and b. Hence, in this model, there are continuous set of static background spacetimes which solve the cosmological constant problem and the gauge hierarchy problem together, specified by, for example, two parameters $(k/M, kb)$ and k_0 and $k_{\frac{1}{2}}$ can be expressed in terms of them from (45) and (52)

$$\bar{k}_0 = \frac{\sinh(kb) - 10^{32}\,(k/M)}{\cosh(kb) - 1}, \qquad (53)$$

$$\bar{k}_{\frac{1}{2}} = \frac{-\bar{k}_0 + \tanh(kb)}{1 - \bar{k}_0\tanh(kb)}. \qquad (54)$$

The original RS model with $k = -k_0 = k_{\frac{1}{2}}$ is a special case where p_5 in (47) vanishes.

Note that while the hidden scale is larger than the weak scale by the ratio of warp factors at two branes, $n(\tau, \frac{1}{2})/n(\tau, 0)$, it is in general different

from the four-dimensional Planck scale by a factor M/k and a different warp factor combination. In the original RS model where $k = -k_0 = k_{\frac{1}{2}}$, $e^{kb} \gg 1$ and $M/k \sim 1$, the difference disappears. If we introduce another hierarchy $M/k \gg 1$ (without any feasible reason at the moment), this will show up in the hierarchy of the hidden scale and the Planck scale.

4.3 One brane model

Let us turn to the cases where matters are added on the brane. First, we look at the case $\rho_{\frac{1}{2}} = 0$. This corresponds to compactifying the extra dimension without the second brane. This is made possible by the tuned $\hat{p}_5$ distribution along the extra dimension

$$\hat{p}_5 = \frac{a_0^3}{na^3}\left[6M^3k^2 - \frac{1}{2}k\coth(kb)(\rho_0 - 3p_0) - \frac{\rho_0(\rho_0 + 3p_0)}{12M^3}\right]. \qquad (55)$$

The 3-brane Friedmann equation (40) becomes

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{K}{a_0^2} = 2k^2\left[-1 + \bar{\rho}_0\coth(kb)\right]. \qquad (56)$$

If we split out the brane tension from the brane energy density, the above equation takes the form of four-dimensional Friedmann equation

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{K}{a_0^2} = 2k^2\left[\left(\frac{k_0}{k}\coth(kb) - 1\right) + \coth(kb)\bar{\rho}_{0M}\right], \qquad (57)$$

with the identifications

$$M_p^2 = \frac{M^3}{k}\tanh(kb), \qquad (58)$$

$$\Lambda_{\rm eff} = \Lambda_0 - (6M^3\Lambda_b)^{\frac{1}{2}}\tanh(kb). \qquad (59)$$

An interesting characteristic of this model is its implication for the cosmological constant problem. Suppose that we have the relation $k = k_0$ in some way, that is, assume a solution to the 'big' cosmological constant problem. Then the size of the cosmological constant has an exponential dependence on the size of extra dimension. This is just the RS-type solution to the 'small' cosmological constant problem. The currently observed cosmological constant $\Omega_\Lambda \sim 1$ can be fitted with $kb \approx 140$.

4.4 Two brane model

Now we turn to the two brane case. We split the brane energy densities into the brane tensions and the brane matter energy densities as we did in the one brane model, and impose the vanishing cosmological constant condition (45). The the 3-brane Friedmann equation (40) can be written as

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{K}{a_0^2} = \frac{\rho_{0M} + \tilde{\rho}_{\frac{1}{2}M} + \dfrac{\rho_{0M}\tilde{\rho}_{\frac{1}{2}M}}{6M^3k}\,\dfrac{\tanh(kb)}{1-\bar{k}_0\tanh(kb)}}{3M_P^2\left[1 - \dfrac{\tilde{\rho}_{\frac{1}{2}M}}{6M_P^2k^2}\,\dfrac{\tanh(kb)\tanh(kb/2)}{1-\bar{k}_0\tanh(kb)}\right]}, \tag{60}$$

where $\tilde{\rho}_{\frac{1}{2}M} = \rho_{\frac{1}{2}M} n(\frac{1}{2})^4 = \rho_{\frac{1}{2}M}\left[\cosh(kb) - \bar{k}_0\sinh(kb)\right]^2$, which is the physically observed energy density on the hidden brane. Up to small correction, this equation is nothing but the four-dimensional Friedmann equation. The energy densities on both branes contribute equally. So the matter on the hidden brane acts as dark matter for our brane.

The inclusion of the effect of stabilization mechanism, through the balanced $\hat{T}^5_5$ component in this simplified model, gives a ordinary FRW cosmology at least up to TeV scale, resolving the peculiarities caused by the existence of the negative tension brane. Above the TeV scale, we meet a complicated situation where in addition that the quadratic terms become important, we need to consider the excitation of other dynamical variables which may spoil the stabilization. Further study is required to clarify it.

5 Conclusion

There are many interesting issues in the brane world models and large/warped extra dimensions, such as the mass hierarchy, the cosmological constant, localization of gravity, confinement of fields on the brane, fine tunings of the bulk cosmological constant and brane tensions, stabilization of the extra dimension, the role of supersymmetry, the role of AdS/CFT duality, connection to string theory, and collider signals, etc.

In this talk, we focused on the cosmological implications, together with the cosmological constant and the gauge hierarchy. In cosmological side, the model with one positive tension brane and the infinite warped extra dimension (RS2) has a viable cosmology without the need of stabilization. But for models with two or more branes or with the compact extra dimension, taking the effects of stabilization into account is very crucial in studying the cosmology of the models. We have shown that, through the simple method using the balaced $\hat{T}^5_5$ component, the inclusion of the effects of stabilization recovers the

conventional four-dimensional FRW cosmology. Therefore, the stabilized RS models have viable cosmology below TeV scale, with interesting new perspective on the mass hierarchy and the cosmological constant. Further study is required to clarify the cosmology of these models at temperatures above TeV scale.

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